Photonische Kristalle

für die integrierte optische Quantentechnologie

Tim Kroh am 04.07.2011

Seminar Optik/Photonik

Motivation

- Quanteninformationstechnologie
- universeller Quantencomputer
  → benötigt universellen Quantengattersatz
- Photonen als flying Qubits
  → intensive Wechselwirkung!

Gliederung

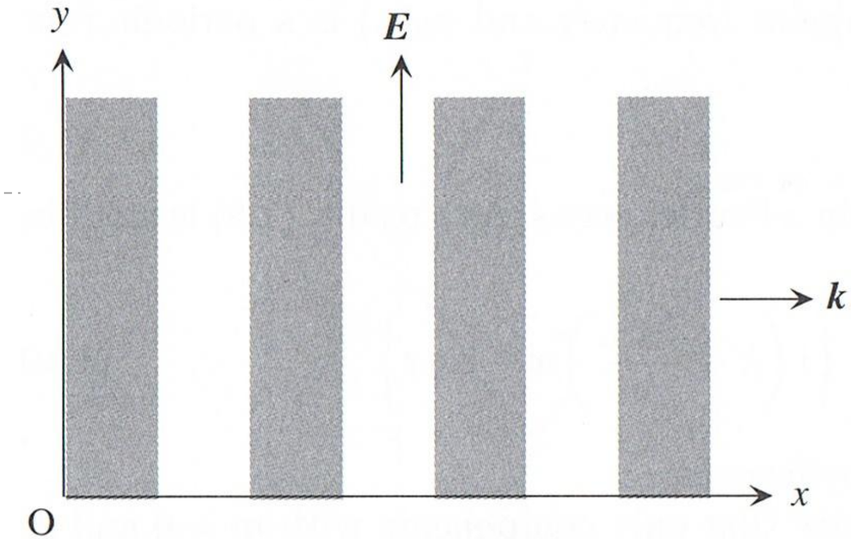
- 1. Photonische Kristalle
  - Grundprinzip in 1D
  - 3D und 2D photonische Kristalle
  - Defekte im 2D photonischen Kristall
- 2. Resonator QED
  - Jaynes-Cummings-Modell
- 3. Kontrolliertes 2-Qubit Phasengatter

1. Photonische Kristalle
Grundprinzip in 1D

1D photonischer Kristall

- Wellengleichung in 1D:

$$\frac{c^2}{\varepsilon(x)}\frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial t^2}$$



[*Optical Properties of Photonic Crystals;* K. Sakoda; Springer (2001)]

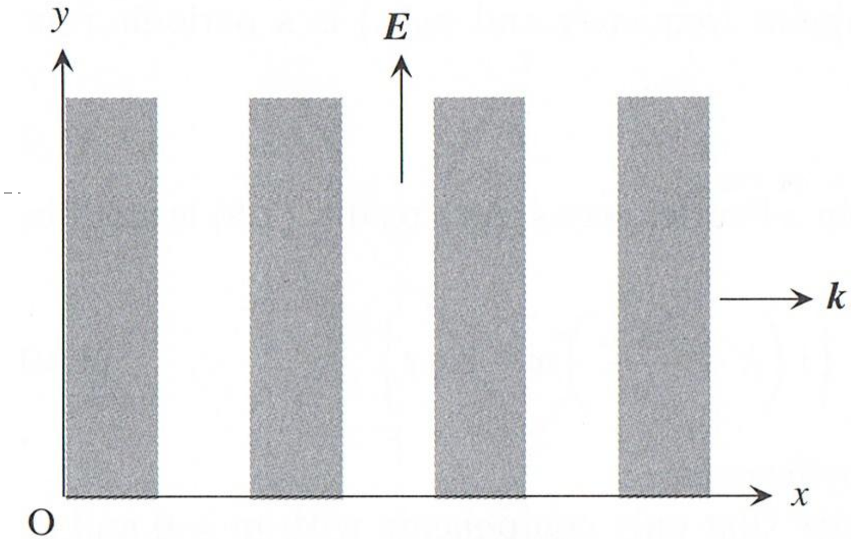
mit der periodischen relativen Permittivität

$$\varepsilon(x) = \varepsilon(x+a)$$

→Fourierzerlegung $\varepsilon^{-1}(x) = \sum_{m=-\infty}^{+\infty} \kappa_m \exp\left(\mathrm{i}\frac{2\pi m}{a}x\right)$

$$\varepsilon^{-1}(x) \approx \kappa_{-1}\mathrm{e}^{-\mathrm{i}\frac{2\pi}{a}x} + \kappa_0 + \kappa_{+1}\mathrm{e}^{+\mathrm{i}\frac{2\pi}{a}x}$$

1D photonischer Kristall



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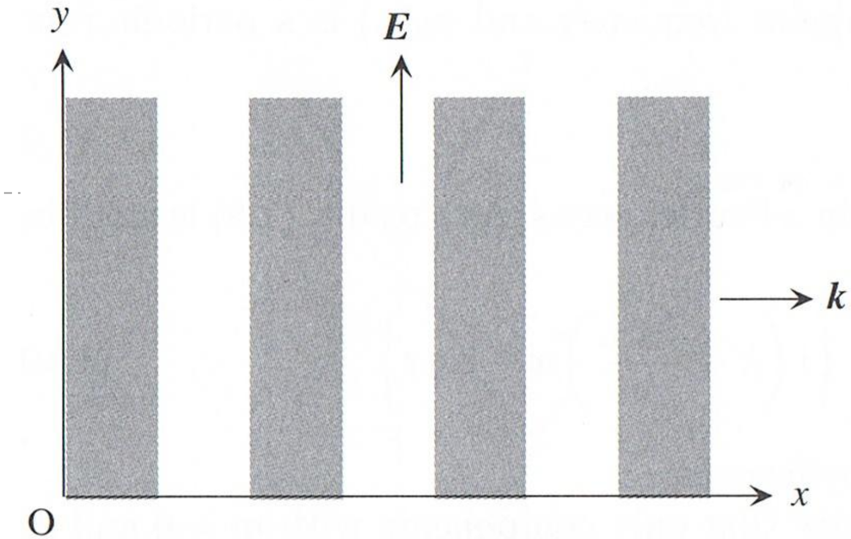
- Wellengleichung in 1D:

$$\frac{c^2}{\varepsilon(x)}\frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial t^2}$$

- Ansatz: $E(x,t) \equiv E_k(x,t) = u_k(x)\exp\{\mathrm{i}(kx-\omega_k t)\}$

$$E_k(x,t) = \sum_{m=-\infty}^{+\infty} E_m \exp\left(\mathrm{i}\frac{2\pi m}{a}x\right)\exp\{\mathrm{i}(kx-\omega_k t)\}$$

1D photonischer Kristall



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- Ansatz: $E(x,t) \equiv E_k(x,t) = u_k(x)\exp\{\mathrm{i}(kx - \omega_k t)\}$

$$E_k(x,t) = \sum_{m=-\infty}^{+\infty} E_m \exp\left\{\mathrm{i}\left(\frac{2\pi m}{a} + k\right)x - \mathrm{i}\omega_k t\right\}$$

1D photonischer Kristall

$$\frac{c^2}{\varepsilon(x)}\frac{\partial^2 E}{\partial x^2}=\frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E_k}{\partial t^2}=(-\mathrm{i}\omega_k)^2\sum_{m=-\infty}^{+\infty}E_m\exp\left\{\mathrm{i}\left(\frac{2\pi m}{a}+k\right)x-\mathrm{i}\omega_k t\right\}$$

$$\frac{c^2}{\varepsilon(x)}\frac{\partial^2 E_k}{\partial x^2}\approx c^2\left(\kappa_{-1}\mathrm{e}^{-\mathrm{i}\frac{2\pi}{a}x}+\kappa_0+\kappa_{+1}\mathrm{e}^{+\mathrm{i}\frac{2\pi}{a}x}\right)\cdot$$

$$\cdot\sum_{m=-\infty}^{+\infty}\left(i\left(\frac{2\pi m}{a}+k\right)\right)^2 E_m\exp\left\{\mathrm{i}\left(\frac{2\pi m}{a}+k\right)x-\mathrm{i}\omega_k t\right\}$$

1D photonischer Kristall

$$\frac{c^2}{\varepsilon(x)}\frac{\partial^2 E}{\partial x^2}=\frac{\partial^2 E}{\partial t^2}$$

$$E_{m-1}\kappa_{+1}\left\{k+\frac{2(m-1)\pi}{a}\right\}^2+E_{m+1}\kappa_{-1}\left\{k+\frac{2(m+1)\pi}{a}\right\}^2$$

$$\approx E_m\left\{\frac{{\omega_k}^2}{c^2}-\kappa_0\left(k+\frac{2m\pi}{a}\right)^2\right\}$$

- für $m=0$ und $m=-1$:

$$\{{\omega_k}^2-\kappa_0 c^2 k^2\}E_0-\kappa_{+1}c^2\left(k-\frac{2\pi}{a}\right)^2 E_{-1}=0$$

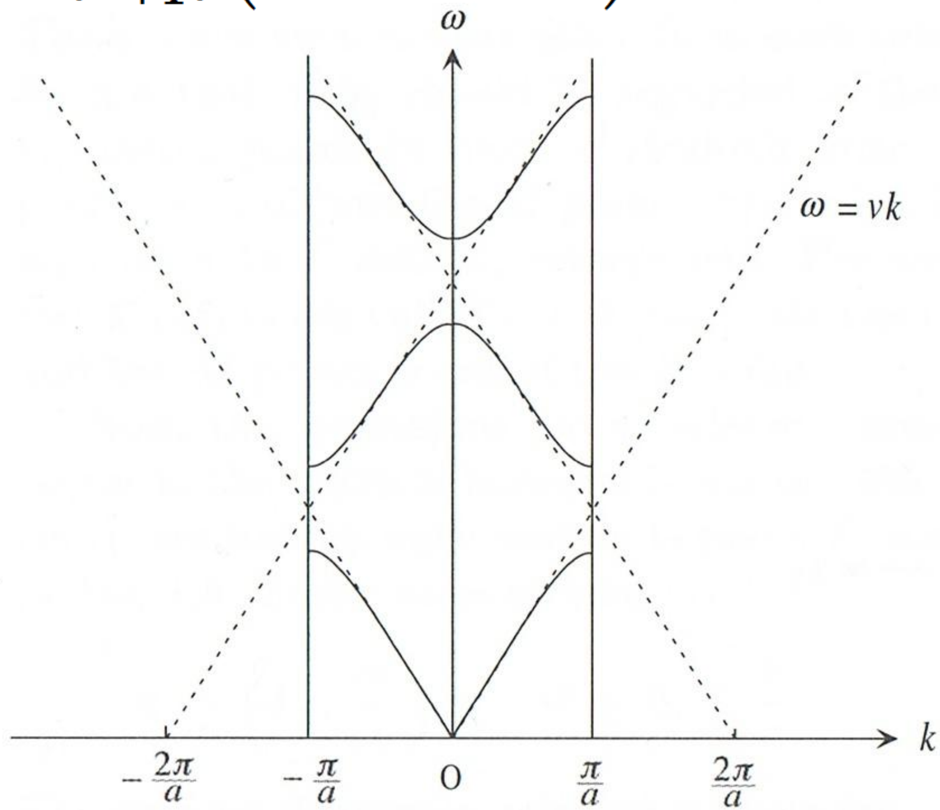
$$-\kappa_{-1}c^2k^2E_0+\left\{{\omega_k}^2-\kappa_0 c^2\left(k-\frac{2\pi}{a}\right)^2\right\}E_{-1}=0$$

1D photonischer Kristall

$$\frac{c^2}{\varepsilon(x)}\frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial t^2}$$

$$\omega_{k,\pm} \approx \frac{\pi c}{a}\sqrt{\kappa_0 \pm |\kappa_{+1}|} \pm \frac{ac}{\pi|\kappa_{+1}|}\left(\kappa_0{}^2 - \frac{|\kappa_{+1}|^2}{2}\right)h^2$$

$$h = k - \pi/a$$



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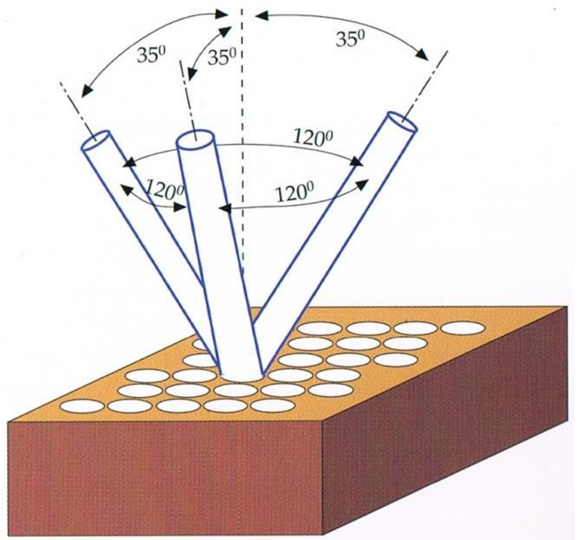
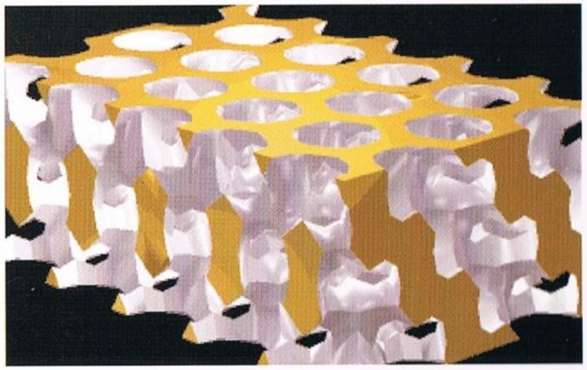
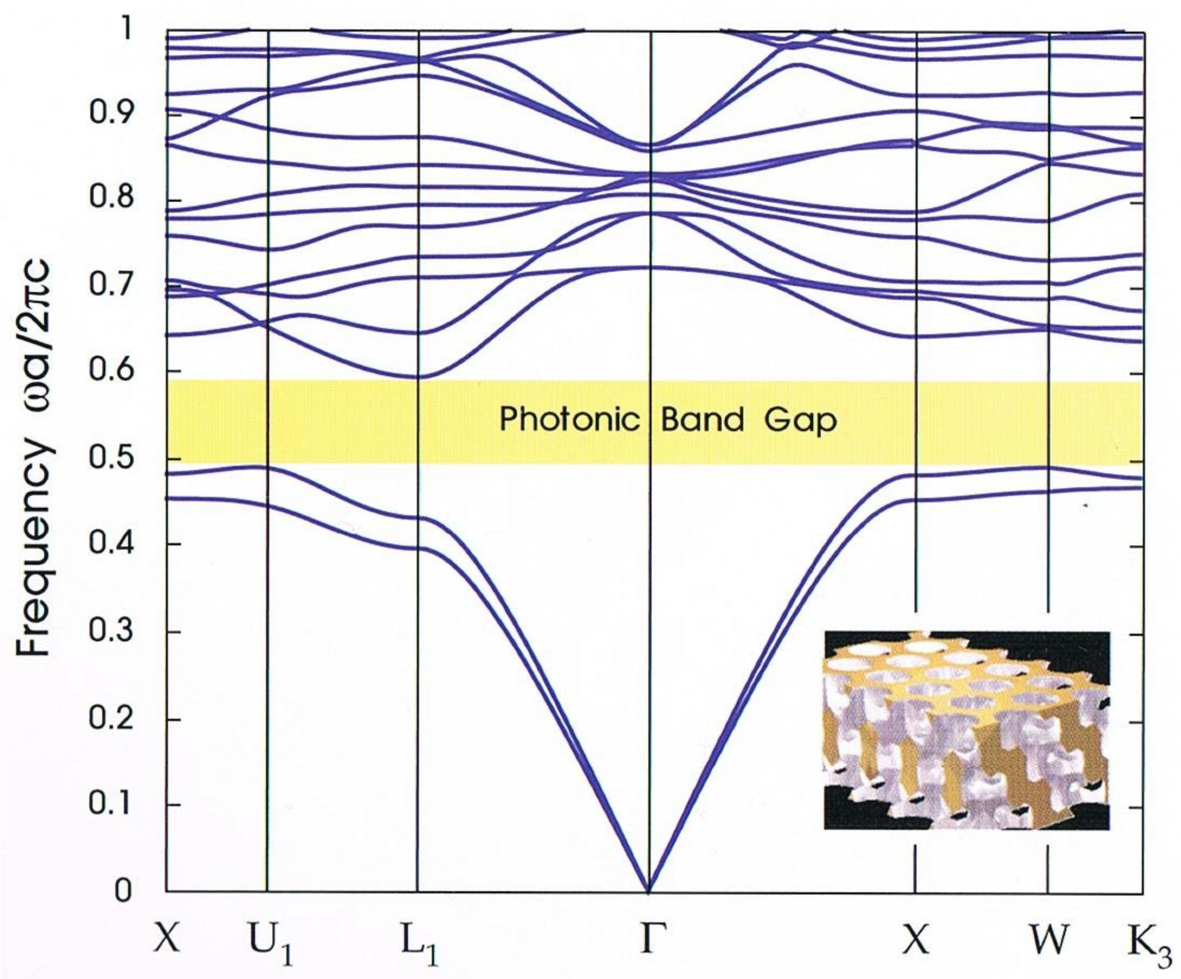
1. Photonische Kristalle
3D und 2D photonische Kristalle

3D photonischer Kristall

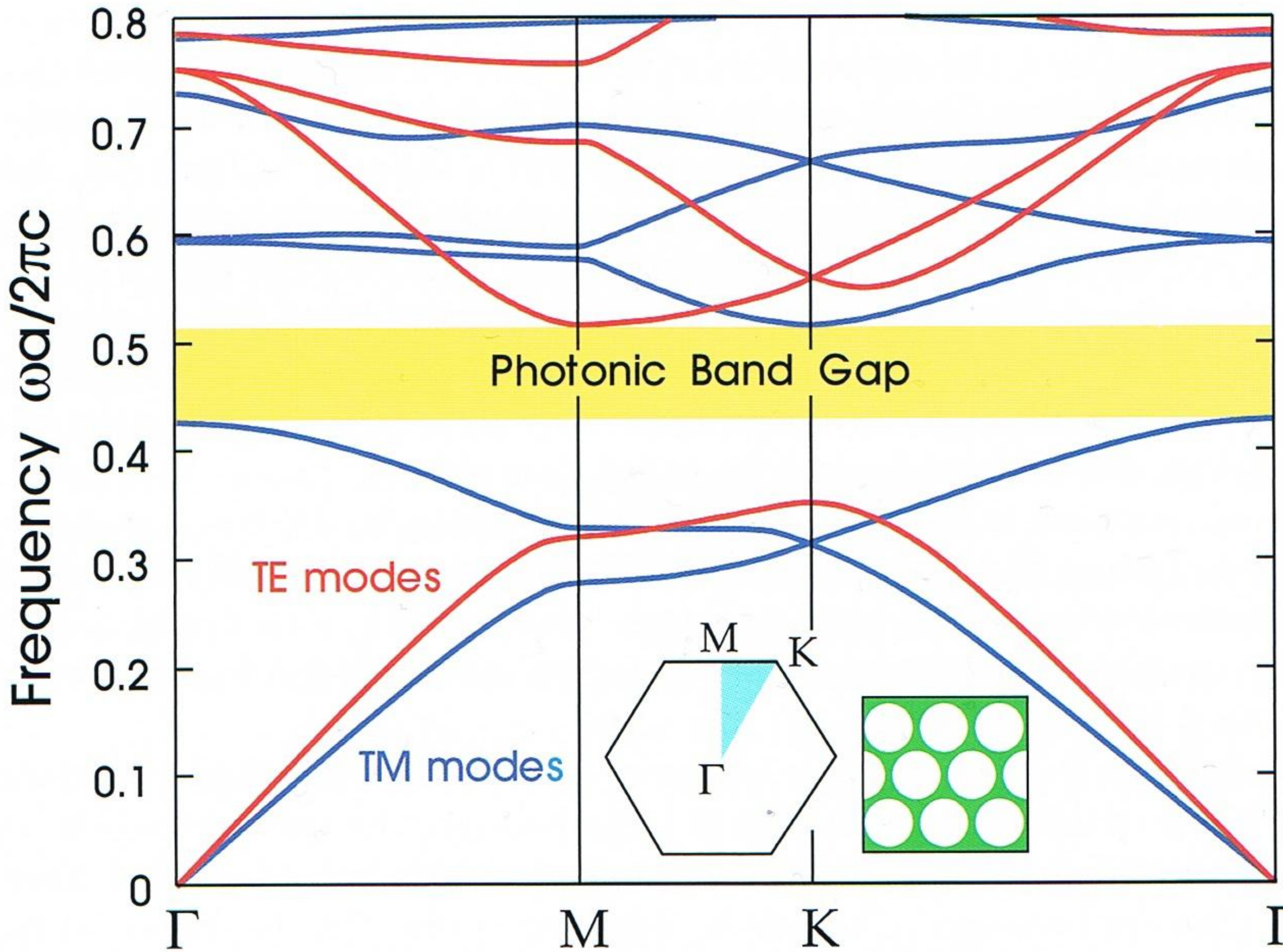






[*Photonic Crystals: Molding the Flow of Light;* J. D. Joannopoulos, S. G. Johnson, J. N. Winn, R. D. Meade; Princeton University Press (2008)]

2D photonischer Kristall

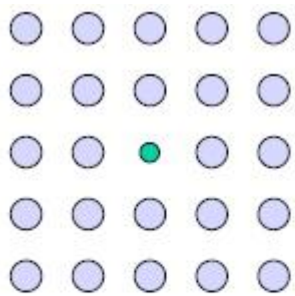


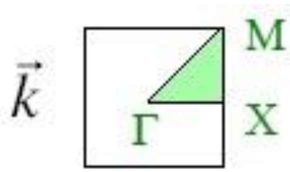
[*Photonic Crystals: Molding the Flow of Light;* J. D. Joannopoulos, S. G. Johnson, J. N. Winn, R. D. Meade; Princeton University Press (2008)]

1. Photonische Kristalle
Defekte im 2D photonischen Kristall

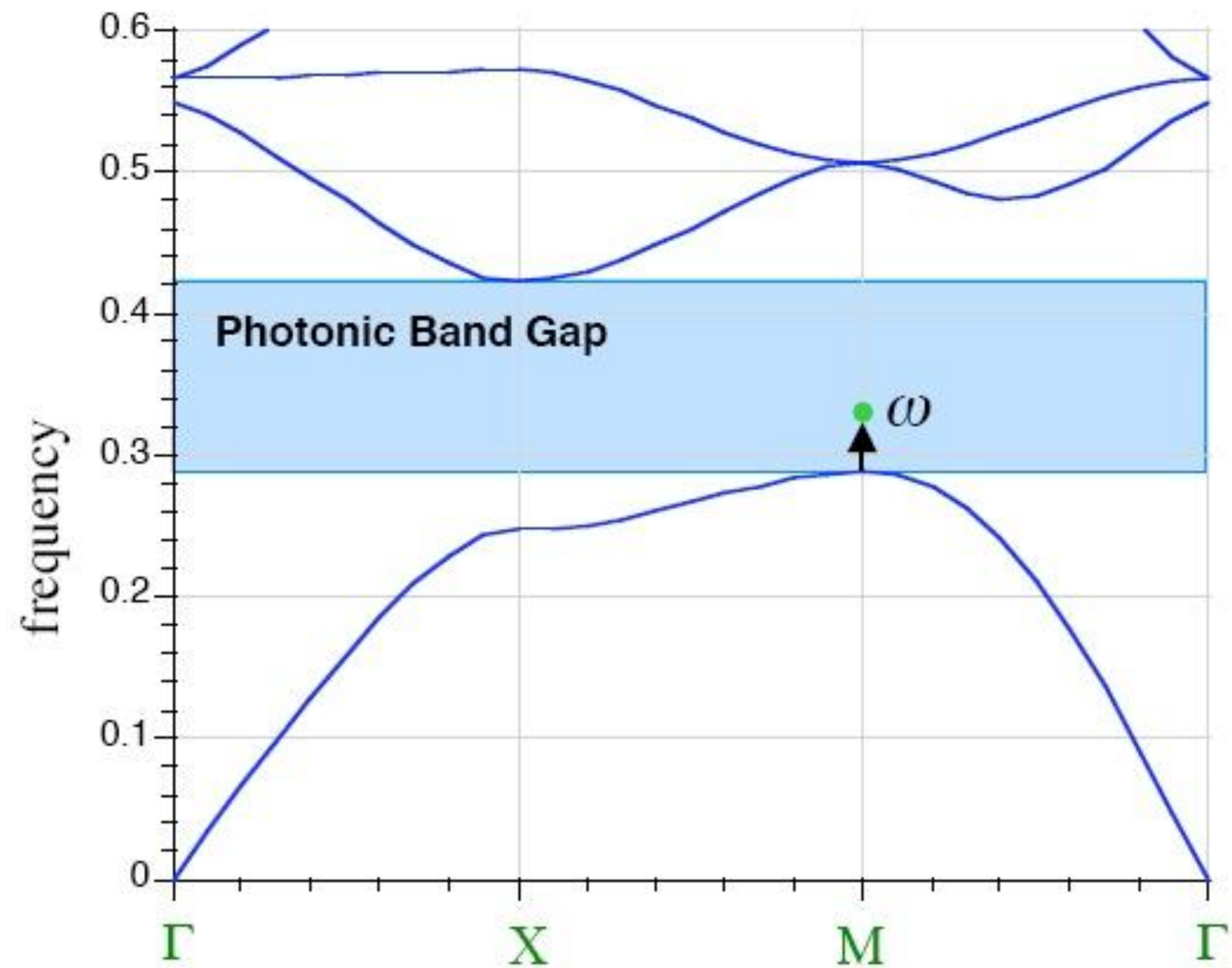
Defekte im 2D photonischen Kristall

- Punktdefekt





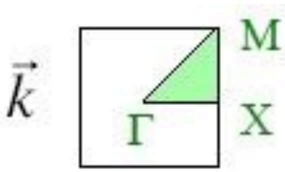
- Resonator

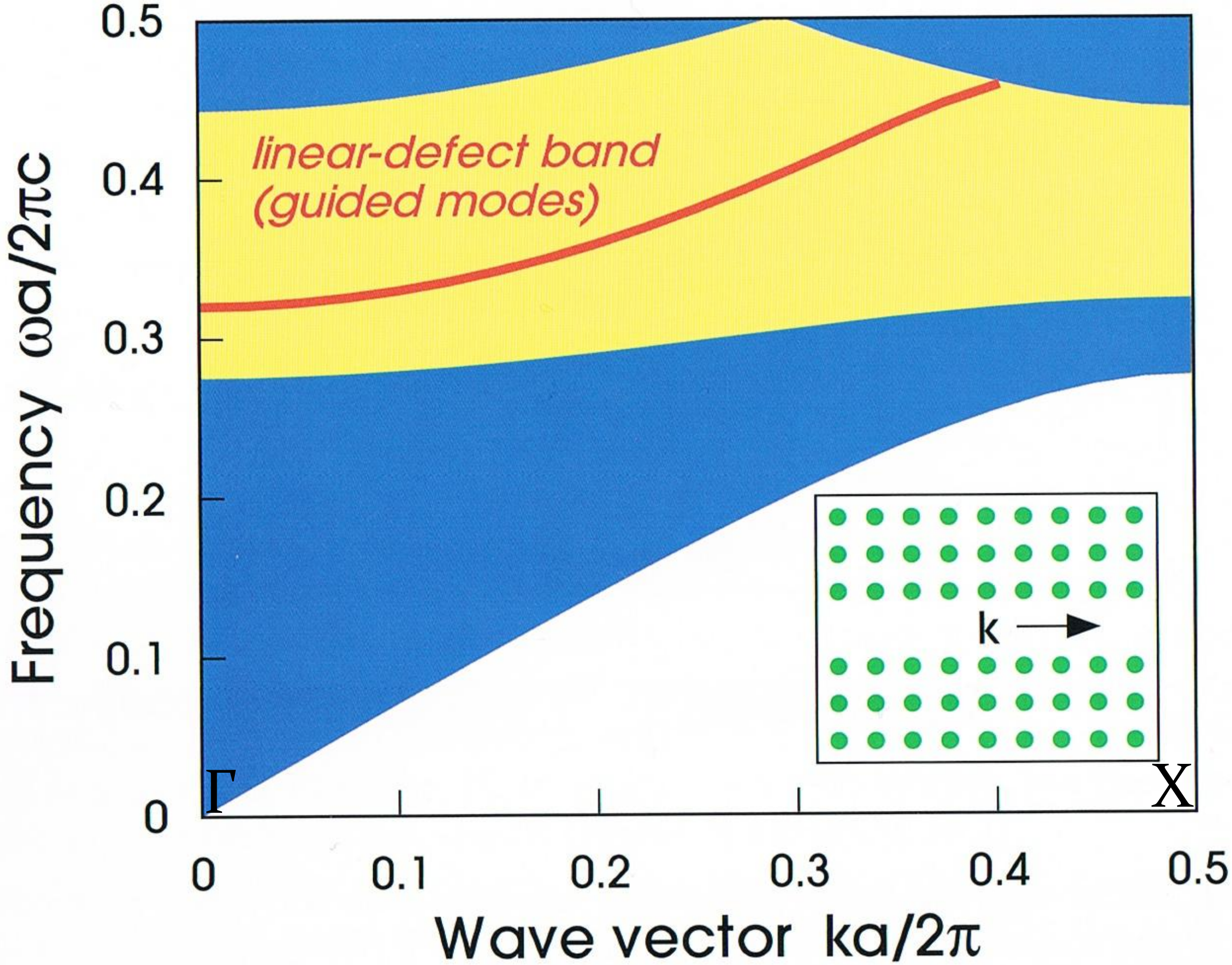


[http://ab-initio.mit.edu/photons/tutorial/L2-defects.pdf (Stand: 03.07.2011)]

Defekte im 2D photonischen Kristall

- Liniendefekt





- Waveguide

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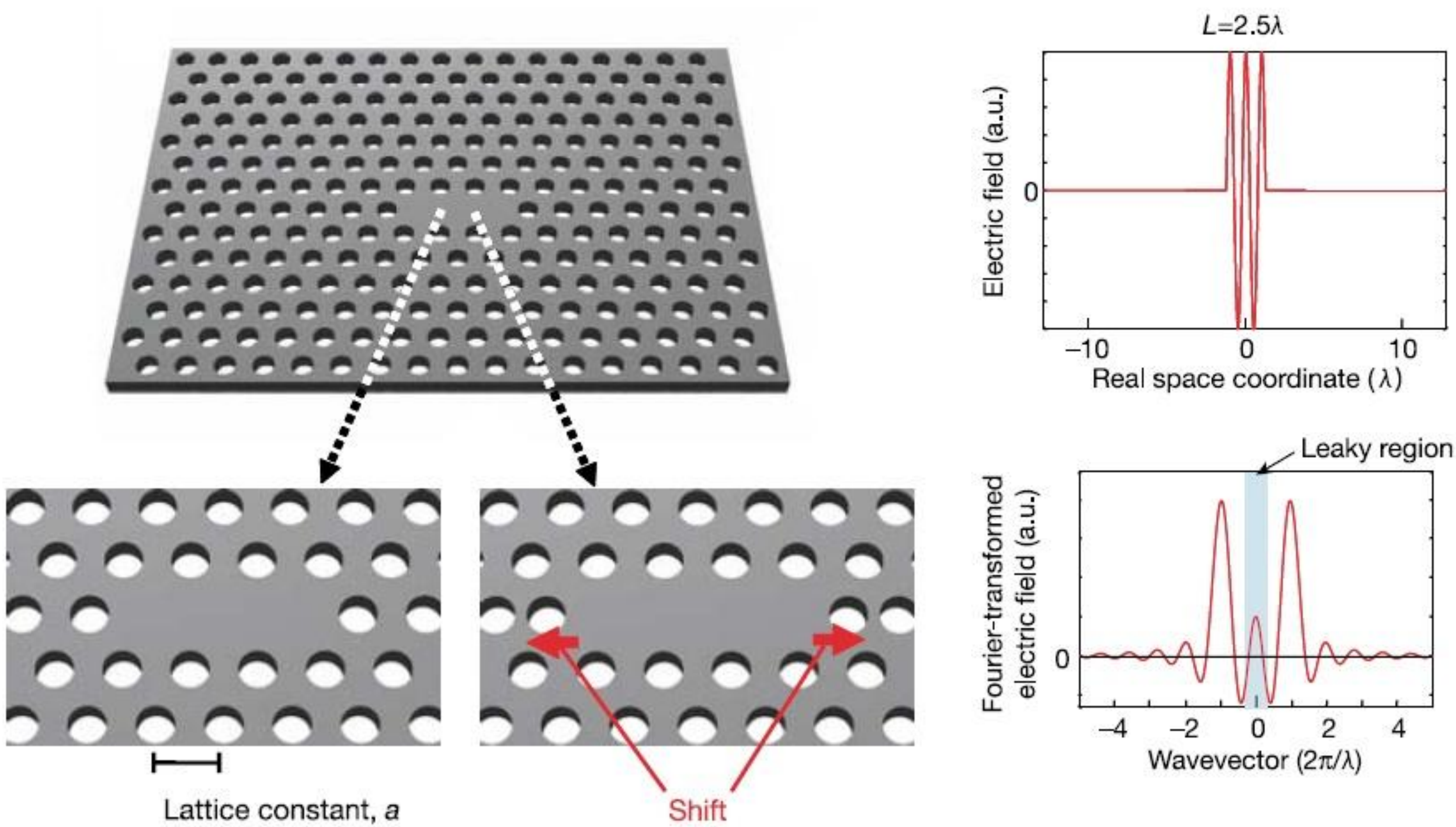
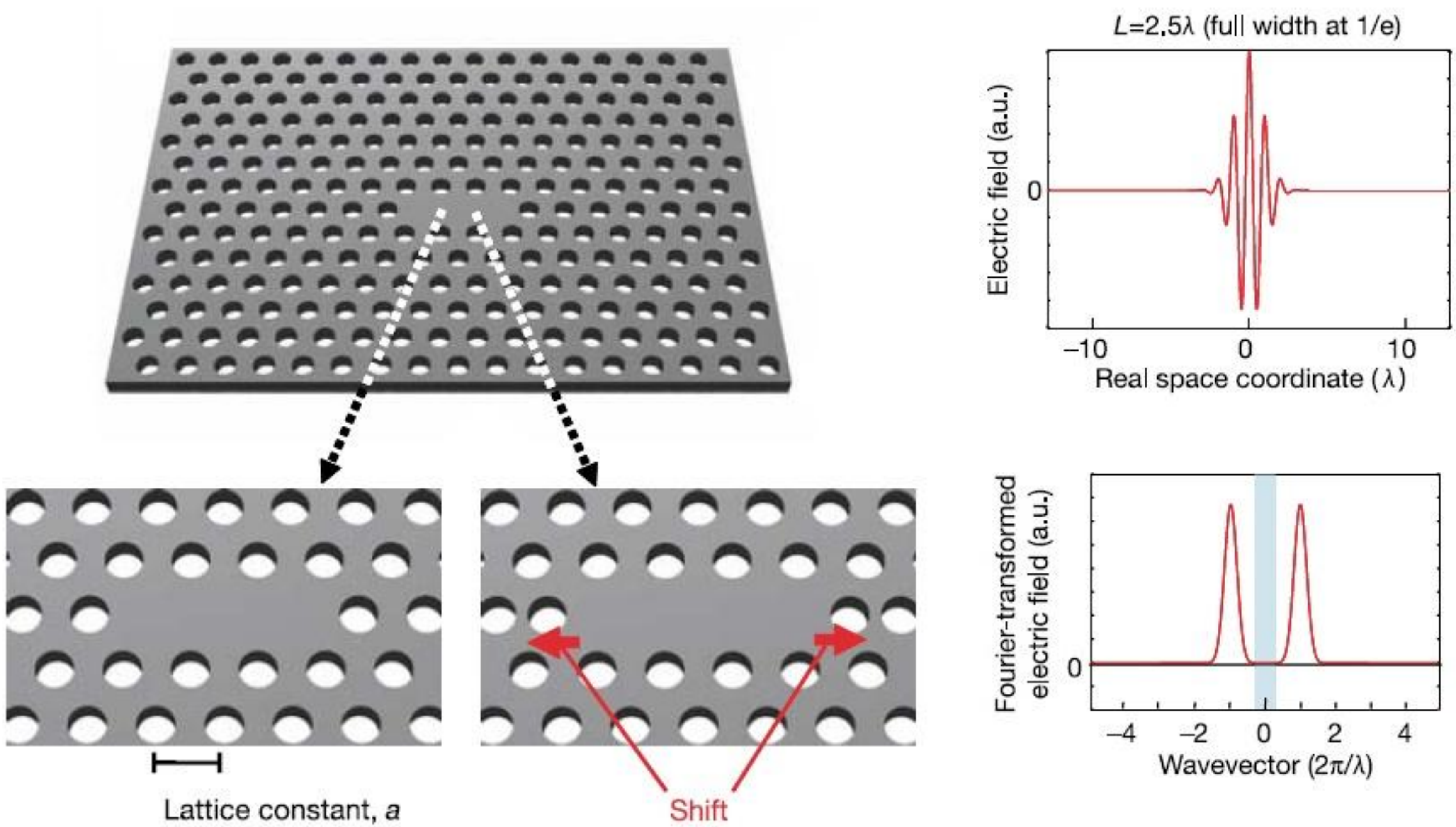
Defekte im 2D photonischen Kristall



[*High-Q photonic nanocavity in a two-dimensional photonic crystal;* Y. Akahane, T. Asano, B.-S. Song, S. Noda; Nature, Volume **425**, 944-947 (2003)] (bearbeitet)

Defekte im 2D photonischen Kristall



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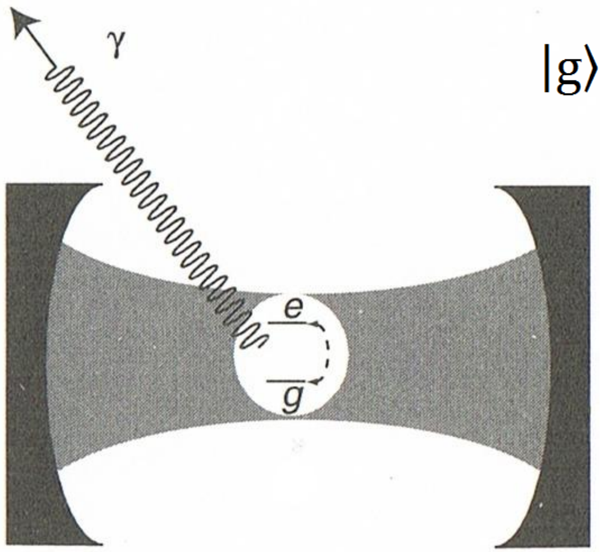
2. Resonator QED
Jaynes-Cummings-Modell

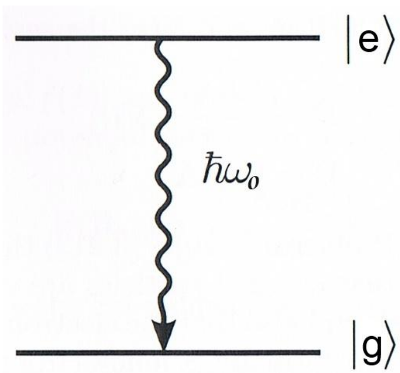
Jaynes-Cummings-Modell

$$|\mathrm{g}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad |\mathrm{e}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\hat{H}_{\mathrm{JC}} = \hat{H}_{\mathrm{Feld}} + \hat{H}_{\mathrm{Atom}} + \hat{H}_{\mathrm{WW}}$$



[*Quantum Optics;* D. F. Walls, G. J. Milburn; Springer (2008)] (bearbeitet)



[*Elements of Quantum Optics;* P. Meystre, M. Sargent III; Springer (2010)] (bearbeitet)

- in Drehwellennäherung

(rotating wave approximation):

$$\hat{H}_{\mathrm{JC}} = \hbar\omega \hat{a}^{\dagger}\hat{a} + \frac{\hbar\omega_0}{2}\hat{\sigma}_z + g\left(\hat{a}^{\dagger}\hat{\sigma}_- + \hat{a}\hat{\sigma}_+\right)$$

$$\text{mit } \hat{\sigma}_{\pm} = \frac{\hat{\sigma}_x \pm \mathrm{i}\hat{\sigma}_y}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \text{ bzw. } \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

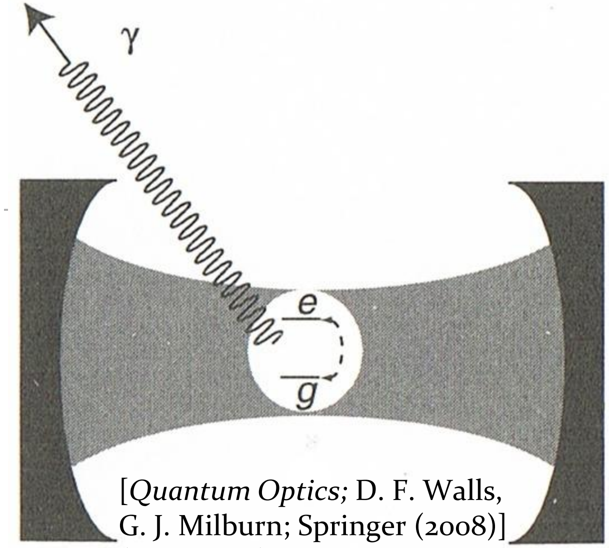
$$\hat{\sigma}_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$$

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Jaynes-Cummings-Modell

$$\hat{H}_{\text{JC}} = \hbar\omega\hat{a}^\dagger\hat{a} + \frac{\hbar\omega_0}{2}\hat{\sigma}_z + g\left(\hat{a}^\dagger\hat{\sigma}_- + \hat{a}\hat{\sigma}_+\right)$$

$$|\text{g}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |0\rangle \;, \quad |\text{e}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |1\rangle$$



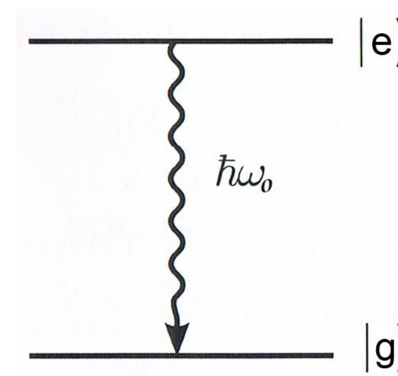
[*Quantum Optics;* D. F. Walls, G. J. Milburn; Springer (2008)] (bearbeitet)

- eine Anregung (Photon) im Resonator

→ Basiszustände $|1\rangle_{\text{Photon}}|0\rangle_{\text{Atom}}$

$|0\rangle_{\text{Photon}}|1\rangle_{\text{Atom}}$

$|0\rangle_{\text{Photon}}|0\rangle_{\text{Atom}}$

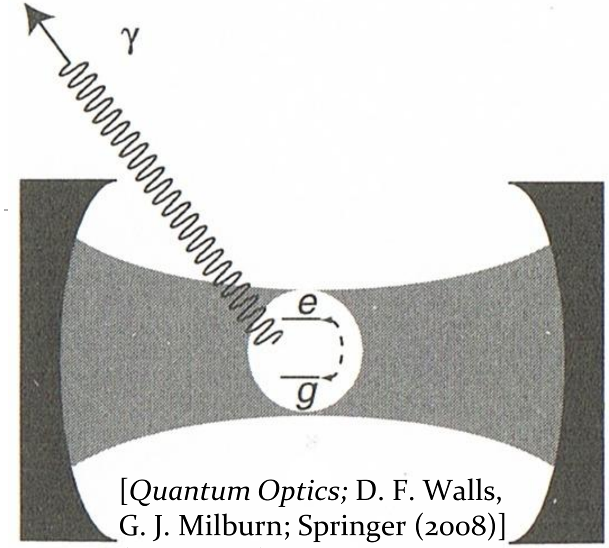


[*Elements of Quantum Optics;* P. Meystre, M. Sargent III; Springer (2010)] (bearbeitet)

Jaynes-Cummings-Modell

$$\hat{H}_{\mathrm{JC}} = \hbar\omega\hat{a}^{\dagger}\hat{a} + \frac{\hbar\omega_0}{2}\hat{\sigma}_z + g\left(\hat{a}^{\dagger}\hat{\sigma}_- + \hat{a}\hat{\sigma}_+\right)$$

- Kopplungsstärke $g \sim \frac{|\vec{d}\vec{E}|}{\hbar}$



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$$\tilde{V} = \frac{\int_V \varepsilon(\vec{r})\left|\vec{E}(\vec{r})\right|^2 \mathrm{d}^3 r}{\left[\varepsilon(\vec{r})\left|\vec{E}(\vec{r})\right|^2\right]_{\max}}$$

$$\frac{1}{\tilde{V}} \sim \left[\left|\vec{E}(\vec{r})\right|^2\right]_{\max} \rightarrow \quad g \sim \frac{1}{\sqrt{\tilde{V}}}$$

Jaynes-Cummings-Modell

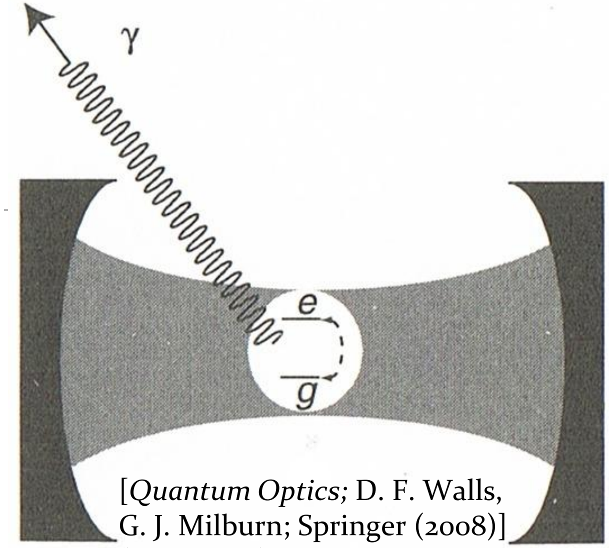
$$\hat{H}_{\mathrm{JC}} = \hbar\omega\hat{a}^{\dagger}\hat{a} + \frac{\hbar\omega_0}{2}\hat{\sigma}_z + g\left(\hat{a}^{\dagger}\hat{\sigma}_- + \hat{a}\hat{\sigma}_+\right)$$

- Kopplungsstärke $g \sim \frac{1}{\sqrt{\tilde{V}}}$
- quality factor Q

$$\mathrm{e}^{-2\pi} = \mathrm{e}^{-\gamma_1 Q}$$

$$\rightarrow \quad \gamma \sim \frac{1}{Q}$$

$$\Longrightarrow \quad \frac{g}{\gamma} \sim \frac{Q}{\sqrt{\tilde{V}}} \quad \text{groß!}$$



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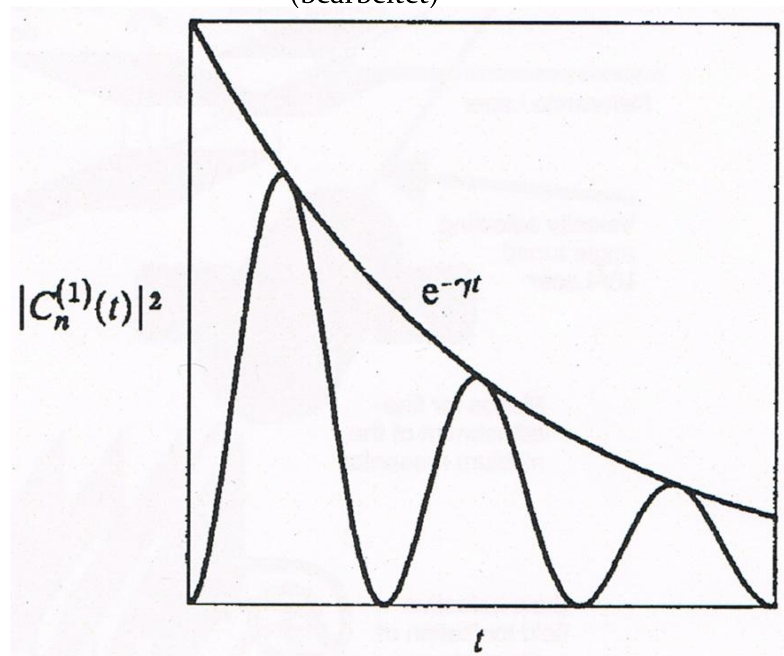
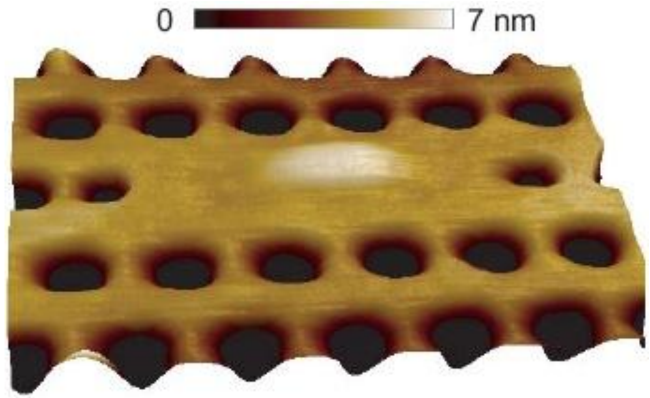


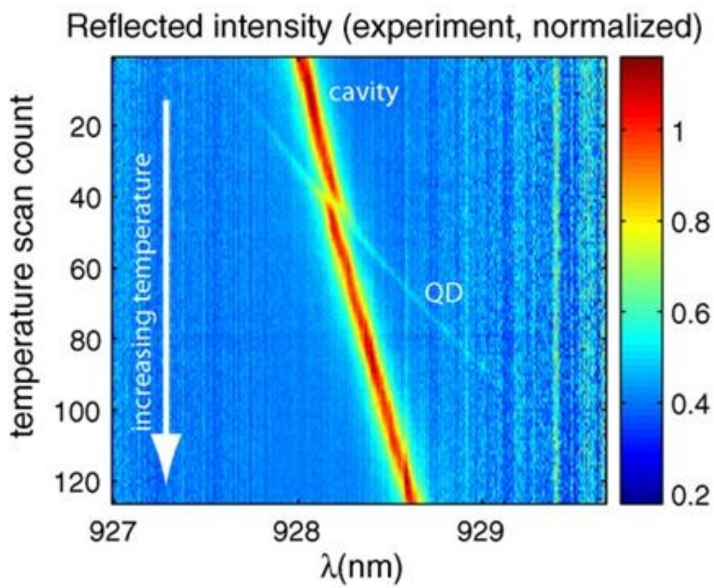
[*Quantum Information & Quantum Computing (Experimental Part);* O. Benson (2009)]

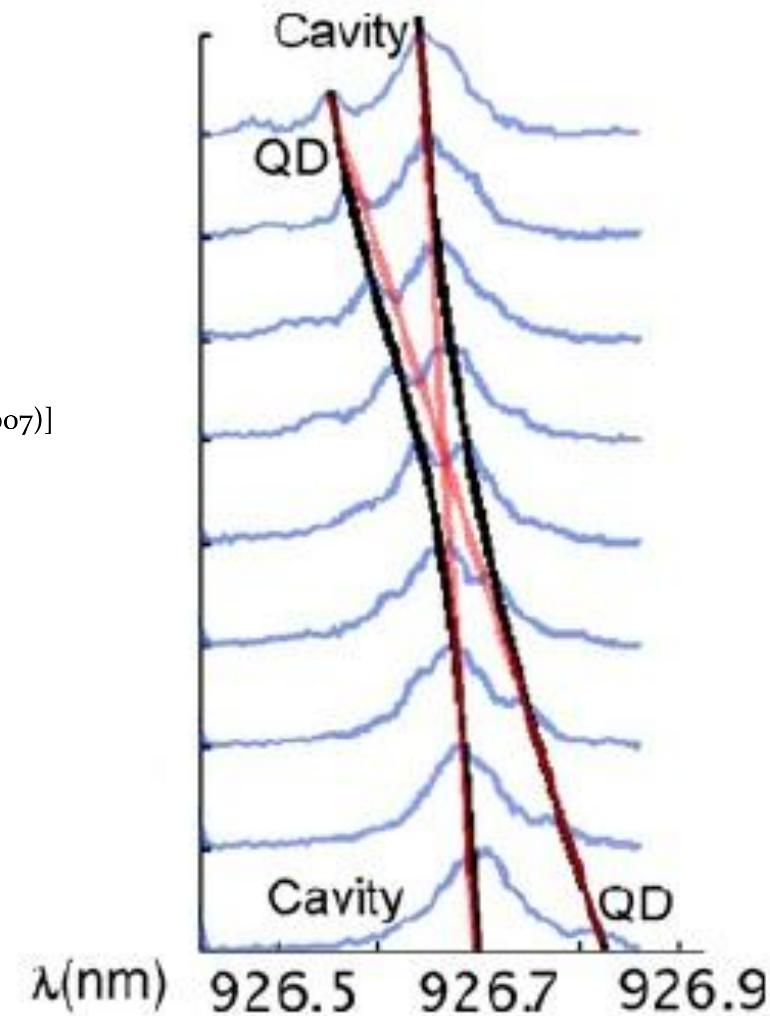
3. Kontrolliertes 2-Qubit Phasengatter

Kopplung von Quantenpunkt und Resonator



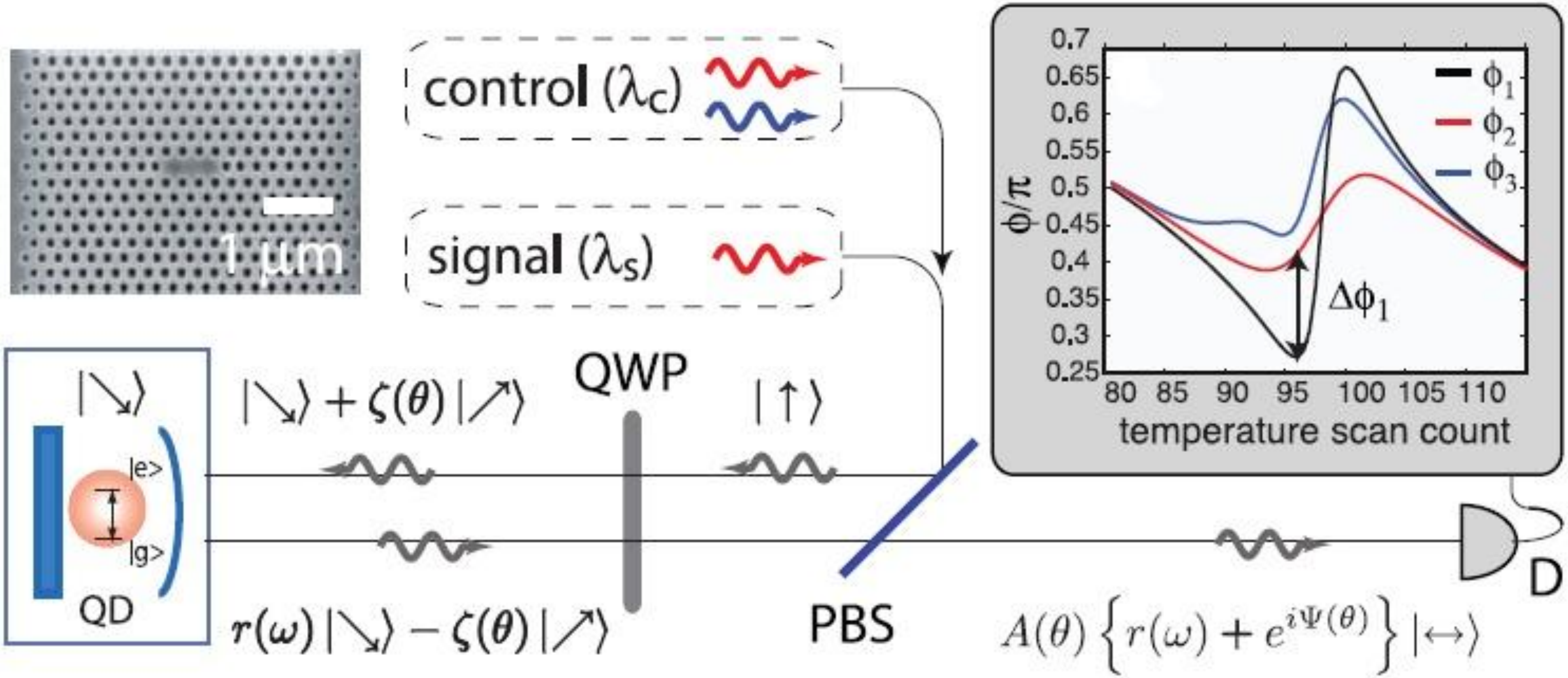
[K. Hennessy, A. Badolato, M. Winger *et al.*; Nature **445** (2007)]





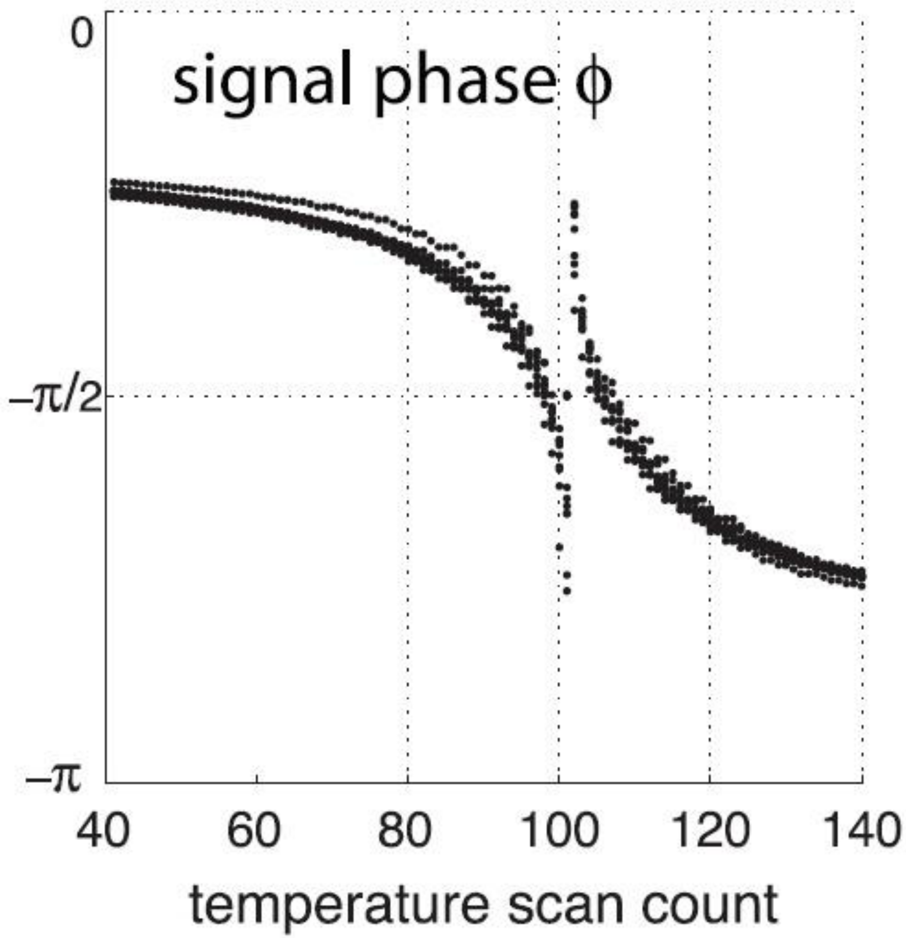
[D. Englund, I. Fushman, A. Faraon, J. Vučković; Photonics and Nanostructures – Fundamentals and Applications **7** (2009)]

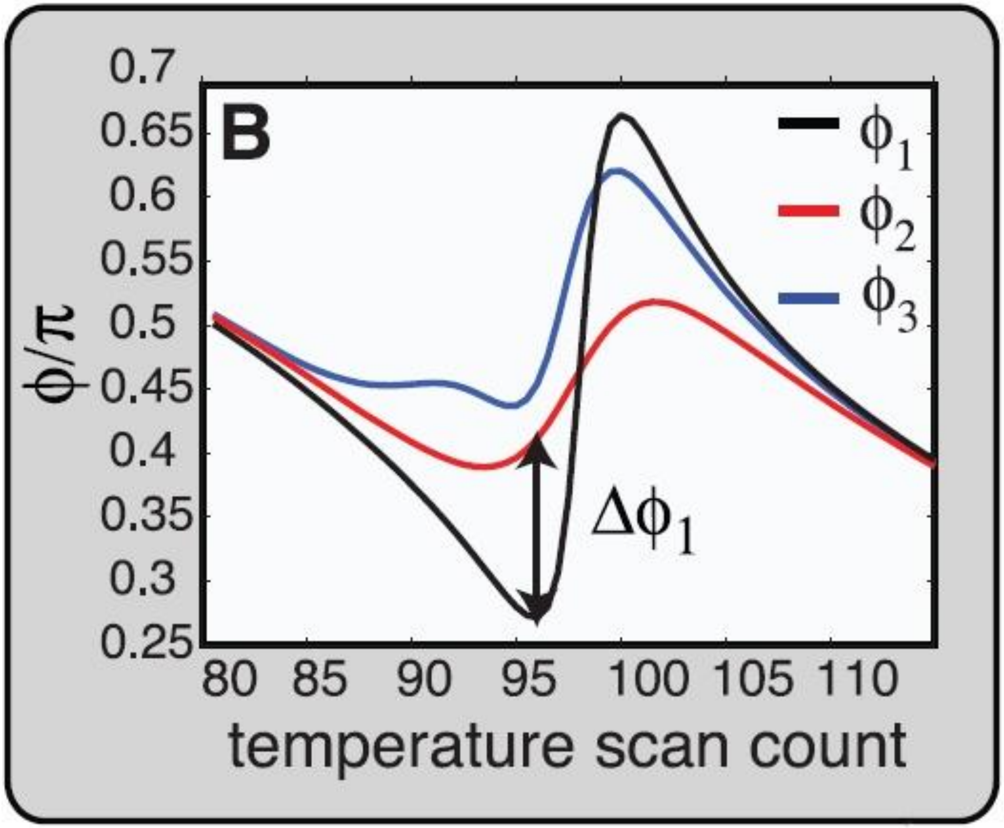
Umsetzung
eines kontrollierten 2-Qubit Phasengatters



[*Controlled Phase Shifts with a Single Quantum Dot;* I. Fushman, D. Englund, A. Faraon, N. Stoltz, P. Petroff, J. Vučković; Science **320** (2008)] (bearbeitet)

Umsetzung
eines kontrollierten 2-Qubit Phasengatters





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Vielen Dank für Ihre Aufmerksamkeit!

Quellen

- *Photonic Crystals: Molding the Flow of Light;* J. D. Joannopoulos, S. G. Johnson, J. N. Winn, R. D. Meade; Princeton University Press (2008)
- *Optical Properties of Photonic Crystals;* K. Sakoda; Springer (2001)
- *Photonic Crystals: The Road from Theory to Practice;* S. G. Johnson, J. D. Joannopoulos; Kluwer Academic Publishers (2003)
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- *Elements of Quantum Optics;* P. Meystre, M. Sargent III; Springer (2010)
- *Quantum Optics;* D. F. Walls, G. J. Milburn; Springer (2008)
- *Quantum Information & Quantum Computing (Experimental Part);* O. Benson (2009)
- *Quantum dots in photonic crystals: From quantum information processing to single photon nonlinear optics;* D. Englund, I. Fushman, A. Faraon, J. Vučković; Photonics and Nanostructures – Fundamentals and Applications, Volume **7**, Issue 1 (2009)

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- *High-Q photonic nanocavity in a two-dimensional photonic crystal;* Y. Akahane, T. Asano, B.-S. Song, S. Noda; Nature, Volume **425**, 944-947 (2003)
- *Controlled Phase Shifts with a Single Quantum Dot;* I. Fushman, D. Englund, A. Faraon, N. Stoltz, P. Petroff, J. Vučković; Science, Volume **320**, 769-772 (2008)
- *Quantum nature of a strongly coupled single quantum dot-cavity system;* K. Hennessy, A. Badolato, M. Winger, D. Gerace, M. Atatüre, S. Gulde, S. Fält, E. L. Hu, A. Imamoğlu; Nature, Volume **445**, 896-899 (2007)
- *Cavity Q, mode volume, and lasing threshold in small diameter AlGaAs microdisks with embedded quantum dots;* K. Srinivasan, M. Borselli, O. Painter, A. Stintz, S.Krishna; Optics Express, Volume **14**, 1094-1105 (2006)
- *Strong-coupling regime for quantum boxes in pillar microcavities: Theory;* L. C. Andreani, G. Panzarini, J.-M. Gérard; Physical Review B, Volume **60**, Number 19 (1999)